NOTATION

0xyz	is the orthogonal coordinate system;
φ_{ij}	is the generalized mean coefficient for the irradiation of the i-th zone by the j-th zone;
F_i	is the zone area;
$\alpha_i, \beta_i, \gamma_i$	are the direction angles of $\bar{n}_i$, the normal to F_i;
abs(p_i)	is the length of radius vector p_i for zone i;
c_i	is the contour of the i-th zone;
r_{ij}	is the distance between points in the i-th and j-th zones;
$\Delta x_{ij}, \Delta y_j, \Delta z_{ij}$	are the projections of r_{ij} on the x, y, and z axes.

LITERATURE CITED

1. V. N. Andrianov, "A new method of optical simulation of radiative heat transfer," Teplofiz. Vys. Temp., 13, No. 4 (1975).
2. R. Siegel and J. R. Howell, Thermal Radiation Heat Transfer, McGraw-Hill (1969).

THERMAL CONDUCTIVITY OF A REINFORCED PLATE

É. Kh. Lokharu and É. A. Tropp

UDC 536.21.01:624.07:517.9

Multiscale expansion is used in asymptotic integration of a steady-state heat-conduction problem for a thin plate of periodic structure; results are presented for the boundary layer near the end.

The singular-perturbation method has proved an efficient means of derving approximate equations for thin bodies; for instance, the method has been applied to the complete equations in the theory of elasticity to derive equations for the bending of a plate [1, 2] or rod [3]. A similar method has been used [4] in the theory of heat conduction for a thermally insulated lateral surface. A method has been given [5, 6] for extending the technique to conditions of the third kind for small values of the Biot number. An approximation has also been constructed [7] for the asymptote to a second boundary-value problem for a second-order elliptic equation of general form for a region in which one dimension is much less than the others.

These studies have envisaged either homogeneous bodies or else bodies in which the parameters vary slowly in space; on the other hand, applications often involve inhomogeneous media in which the parameters vary considerably over distances small by comparison with the length of the body. The simplest case is one where the rapid change is regular, e.g., periodic. Bodies of regular structure are of importance in themselves in the description of reinforced structures [8] as well as in the simulation of irregular inhomogeneous bodies, including random media. The asymptotic methods of [1-3, 5-7] are inadequate for media with rapidly varying parameters. However, another form of the singular-perturbation method, which is widely used in nonlinear mechanics [4], is then effective: two-scale expansion. Here we consider a steady-state problem in the theory of heat conduction for a thin plate reinforced by a rod lattice. It is assumed that the thermal conductivity of the reinforcement differs from that of the matrix material and also that the thermal contact is ideal.

The latter assumption is unimportant for the method given here and is made only in order to simplify the expressions.

Physical considerations show that such a reinforced plate can be replaced approximately by a homogeneous plate whose thermal conductivity along the rod direction is different from that along the transverse direction if we are not interested in the details of the temperature variation over distances small by comparison with the size of the plate in plan. Here we provide a justification for this substitution, i.e., we use the three-dimensional conduction equation to derive a two-dimensional one and construct an algorithm for calculating the corrections to the two-dimensional temperature distribution. This method gives, in particular,

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 3, pp. 497-504, September, 1978. Original article submitted July 25, 1977.

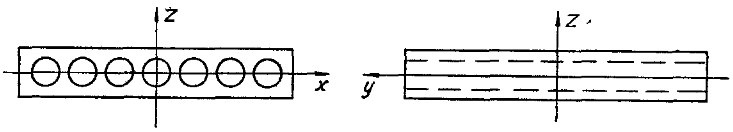

Fig. 1. Cross section of a plate.

asymptotically exact effective thermal conductivities without resort to any hypotheses on the temperature distribution over the thickness of the plate or over the cross section of a rod.

The formulation is as follows. The parallelepiped $-\bar{a} < \bar{x} < \bar{a}$, $-\bar{b} < \bar{y} < \bar{b}$, $-\bar{h} < z < \bar{h}$ ($\bar{h}/\bar{a} \ll 1$, $\bar{h}/\bar{b} \ll 1$) consists of identical blocks adjoining one another (Fig. 1), each of which contains a cylindrical inclusion; the generators of the cylinders are parallel to the $\bar{y}$ axis, while the cross section is a singly coupled region S_1 bounded by a piecewise-smooth curve Γ_1. The characteristic size of a cylinder in cross section is $\bar{d}$, while the pitch of the rod lattice is $\bar{t}$, which is of the same order as the thickness $\bar{h}$ of the plate.

We assume that in the plane $\bar{x} = -\bar{a}$ we have a constant temperature distribution $T(-\bar{a}, \bar{y}, \bar{z}) = F(\bar{y}, \bar{z})$, while the heat transfer occurs in accordance with Newton's law on the other faces (the environmental temperature T_0 is taken as constant). There is ideal contact at the boundary of each inclusion, so the temperature and heat flux are continuous. We then seek to determine the asymptotic solution to Laplace's equation $\partial^2T/\partial\bar{x}^2 + \partial^2T/\partial\bar{y}^2 + \partial^2T/\partial\bar{z}^2 = 0$ when $\varepsilon = \bar{h}/\bar{a} \to 0$, where it is assumed that $\bar{t}/\bar{h}$ and $\bar{d}/\bar{h}$ tend to constant values t and d on passing to the limit.

The two-scale expansion method leads us to replace the dimensional variable $\bar{x}$ by two dimensionless ones: the large-scale variable $x = \bar{x}/\bar{a}$ and the small-scale variable $\xi = \bar{x}/\bar{h}$. The problem is then formulated as follows in terms of the dimensionless variables x, ξ, $y = \bar{y}/\bar{a}$, $z = \bar{z}/\bar{h}$, and $u = (T-T_0)/T_0$: the equation for steady-state heat conduction,

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial z^2} + 2\varepsilon \frac{\partial^2 u}{\partial \xi \partial x} + \varepsilon^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0, \quad (\xi, z) \in S_1 \cup S_2, \ |x| < 1, \ |y| < \delta; \tag{1}$$

the boundary conditions at the outer surface of the plate,

$$u = f(y, z), \ x = -1, \quad \frac{\partial u}{\partial \xi} + \varepsilon \frac{\partial u}{\partial x} = -A\varepsilon^2 u, \ x = 1,$$
$$\frac{\partial u}{\partial y} = \mp g_s A \varepsilon u, \ y = \pm \delta, \quad \frac{\partial u}{\partial z} = \mp A\varepsilon^2 u, \ z = \pm 1; \tag{2}$$

and the conditions at the boundary of an inclusion,

$$[u] = 0, \quad \left[\lambda \frac{\partial u}{\partial n} \right] = \varepsilon (\lambda_1 - \lambda_2) \frac{\partial u}{\partial x} \cos n\xi, \ (\xi, z) \in \Gamma_1. \tag{3}$$

In (1)-(3) we have used the symbols $\delta = \bar{b}/\bar{a}$ and $A = \alpha\bar{a}^2/\lambda\bar{h}$, where α is the heat-transfer coefficient for the side surface, λ_2 is the thermal conductivity of the plate material, and λ_1 is the thermal conductivity of the inclusions. The quantity g_s takes the value 1 if the point lies in region S_1. It has been assumed in writing (2) that the Biot number is of the second order of smallness in ε. It has been shown [5, 6] that this case corresponds to approximately one-dimensional heat conduction for a rod or approximately two-dimensional conduction for a plate.

We assume that as $\varepsilon \to 0$ we have a limiting expansion applicable at some distance from the side faces $x = \pm 1$, $y = \pm\delta$:

$$u = \sum_{k=0}^{\infty} \varepsilon^k u_k.$$

We substitute this series into (1) and use (2) and (3) to obtain the following chain of equations:

$$\frac{\partial^2 u_k}{\partial \xi^2} + \frac{\partial^2 u_k}{\partial z^2} = -2 \frac{\partial^2 u_{k-1}}{\partial x \partial \xi} - \left(\frac{\partial^2 u_{k-2}}{\partial x^2} + \frac{\partial^2 u_{k-2}}{\partial y^2} \right),$$
$$(\xi, z) \in S_1 \cup S_2, \quad \frac{\partial u_k}{\partial z} = \mp A u_{k-2}, \ z = \pm 1, \tag{4}$$

$$[u_k]=0,\ \left[\lambda\frac{\partial u_k}{\partial n}\right]=(\lambda_1-\lambda_2)\frac{\partial u_{k-1}}{\partial x}\cos n\xi,\ (\xi,\ z)\in\Gamma_1. \tag{4}$$

The region defined by the variables ξ and z becomes an infinite band whose structure is periodic along the ξ direction on passing to the limit $\varepsilon \to 0$ with ξ fixed; in the first approximation ($k = 0$), the thermal-insulation condition applies at the bounding planes. The equations and the boundary conditions then have translational symmetry along the ξ direction, with repeat distance t. Consequently, the solution should show the same periodicity. In the subsequent approximations, the right sides of the equations and the boundary condition contain periodic functions of ξ and functions of the slow variable x. The relationship between the variables x and ξ is not explicitly incorporated into the equations involving differentiation with respect to ξ (i.e., the x dependence is parametric), so the functions of x appearing on the right side do not cause a deviation from the translational symmetry in ξ. This means that (4) can be replaced in all approximations by means of the periodicity condition

$$u_k(\xi+t)=u_k(\xi),\ \ \frac{\partial u_k(\xi+t)}{\partial\xi}=\frac{\partial u_k(\xi)}{\partial\xi}. \tag{5}$$

We now solve (4) and (5); these imply for $k = 0$ that u_0 is independent of ξ and z: $u_0 = u_0(x, y)$. All the conditions that determine u_1 are homogeneous, apart from the condition for continuity of the heat flux at the boundary of an inclusion:

$$\left[\lambda\frac{\partial u_1}{\partial n}\right]=(\lambda_1-\lambda_2)\frac{\partial u_0}{\partial x}\cos n\xi,\ (\xi,\ z)\in\Gamma_1. \tag{6}$$

The right side of (6) can be interpreted as a specific density of surface sources; the total output from these sources is proportional to $\int_{\Gamma_1}\cos n\xi\, dl$ and equals zero because the problem for u_1 is soluble. We see from (6) that u_1 should be defined in the form

$$u_1=\frac{\partial u_0}{\partial x}\varphi_1(\xi,\ z)+\psi_1(x,\ y), \tag{7}$$

where φ_1 is defined by the following boundary-value problem:

$$\begin{gathered}\frac{\partial^2\varphi_1}{\partial\xi^2}+\frac{\partial^2\varphi_1}{\partial z^2}=0,\ \ (\xi,\ z)\in S_1\cup S_2,\ \ \frac{\partial\varphi_1}{\partial z}=0,\ \ z=\pm1,\\ \varphi(\xi+t)=\varphi_1(\xi),\ \ \frac{\partial\varphi_1(\xi+t)}{\partial\xi}=\frac{\partial\varphi_1(\xi)}{\partial\xi},\\ [\varphi_1]=0,\ \ \left[\lambda\frac{\partial\varphi_1}{\partial n}\right]=(\lambda_1-\lambda_2)\cos n\xi,\ \ (\xi,\ z)\in\Gamma_1.\end{gathered} \tag{7a}$$

The problem (4), (5) is not soluble for arbitrary right sides of the equations and boundary conditions for $k = 2$; the condition for solubility gives a two-dimensional heat-conduction equation for the unknown function $u_0(x, y)$. The Ostrogradskii–Gauss formula is applied to the left side of (4) for regions I and II, respectively:

$$\lambda_1\iint_{S_1}\Delta u_2 d\xi dz=\lambda_1\int_{\Gamma_1}\frac{\partial u_2}{\partial n}dl,$$

$$\lambda_2\iint_{S_2}\Delta u_2 d\xi dz=-\lambda_2\int_{\Gamma_1}\frac{\partial u_2}{\partial n}dl+\lambda_2\int_{\Gamma_2}\frac{\partial u_2}{\partial n}dl.$$

We add these two equations together and use (4) with (7) to obtain

$$\lambda_1\iint_{S_1}\Delta u_2 d\xi dz+\lambda_2\iint_{S_2}\Delta u_2 d\xi dz=\int_{\Gamma_1}\left[\lambda\frac{\partial u_1}{\partial x}\right]\cos n\xi dl-2\lambda_2 Au_0 t=B(\lambda_1,\ \lambda_2,\ \Gamma_1)\frac{\partial^2 u_0}{\partial x^2}-2\lambda_2 Au_0 t, \tag{8}$$

where $B(\lambda, \lambda_2, \Gamma_1) = \int_{\Gamma_1}\varphi_1(\xi, z)\cos n\xi dl$ is a function of λ_1 and λ_2 and also a functional of Γ_1 (B becomes zero only for $\lambda_1 = \lambda_2$). On the other hand, we can put

$$\lambda_1 \iint\limits_{S_1} \Delta u_2 d\xi dz + \lambda_2 \iint\limits_{S_2} \Delta u_2 d\xi dz = -2\lambda_1 \iint\limits_{S_1} \frac{\partial^2 u_1}{\partial x \partial \xi} d\xi dz -$$

$$-\lambda_1 \iint\limits_{S_1} \Delta_{xy} u_0 d\xi dz - 2\lambda_1 \iint\limits_{S_2} \frac{\partial^2 u_1}{\partial x \partial \xi} d\xi dz - \lambda \iint\limits_{S_2} \Delta_{xy} u_0 d\xi dz.$$

We use (7), Green's formula, and condition (5) to obtain

$$\lambda_1 \iint\limits_{S_1} \Delta u_2 d\xi dz + \lambda_2 \iint\limits_{S_2} \Delta u_2 d\xi dz = 2B \frac{\partial^2 u_0}{\partial x^2} - (\lambda_2 S_2 + \lambda_1 S_1) \Delta_{xy} u_0. \tag{9}$$

We equate the right sides of (8) and (9) to obtain the solubility condition as

$$(\lambda_1 S_1 + \lambda_2 S_2) \left(\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} \right) - B(\lambda_1, \lambda_2, \Gamma_1) \frac{\partial^2 u_0}{\partial x^2} - 2\lambda_2 A t u_0 = 0. \tag{10}$$

This can be interpreted as a two-dimensional heat-conduction equation for a homogeneous anisotropic plate.

The φ_1 of (7a) takes the following form for the particular case of alternating rods of rectangular cross section:

$$\varphi_1 = \begin{cases} -\beta(m-1)(1+\beta m)^{-1}[\xi + (1{,}5\gamma - 1)t], & 0 < \xi < \gamma t, \\ (m-1)(1+\beta m)^{-1}[\xi - 1{,}5\gamma t], & \gamma t < \xi < t, \end{cases}$$

where γt is the width of a rod and $\beta = (1-\gamma)/\gamma$, and $m = \lambda_1/\lambda_2$; we calculate B from the known φ_1 and substitute into (10) to obtain

$$\frac{(1+\beta)^2 m}{(1+\beta m)^2} \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} - \frac{(1+\beta) A}{(1+\beta m)} u_0 = 0. \tag{11}$$

The ratio of the coefficients of the second derivatives corresponds to the ratio of the effective thermal conductivities $\lambda_x = (1+\beta)\lambda_1\lambda_2(\lambda_2 + \beta\lambda_1)^{-1}$ and $\lambda_y = (\lambda_2 + \beta\lambda_1)^{-1}$ as calculated from the theory of chains for serial and parallel connection of conductors, respectively. In general, it is necessary to solve (7a) for the effective thermal conductivities and then to calculate B, which is dependent on λ_1 and λ_2, along with the geometry of the region.

In order to determine the boundary conditions for (10) and to construct a widely suitable expansion one has to add functions of the boundary-layer type to the u_k, as these are localized near the ends $x = \pm 1$, $y = \pm\delta$:

$$u = \sum_{k=0}^{\infty} u_k \varepsilon^k + \sum_{k=0}^{\infty} v_{1k}(\xi_1, y, z)\varepsilon^k + \sum_{k=0}^{\infty} v_{2k}(\xi_2, y, z)\varepsilon^k + \sum_{k=0}^{\infty} w_{1k}(\xi, x, \eta_1, z)\varepsilon^k + \sum_{k=0}^{\infty} w_{2k}(x, \xi, \eta_2, z)\varepsilon^k, \tag{12}$$

where $\xi_{1,2} = (x \pm 1)/\varepsilon$ and $\eta_{1,2} = (y \pm \delta)/\varepsilon$ are functions of boundary-layer type that satisfy the conditions

$$v_i,\ \partial v_i/\partial \xi_i \to 0 \text{ as } \xi_i \to \infty, \quad w_i,\ \partial w_i/\partial \eta_i \to 0 \text{ as } \eta_i \to \infty. \tag{13}$$

We substitute (12) into the equations and boundary conditions and use the conditions for the problem to be solvable for b_0 and w_0 to obtain the following boundary conditions for $u_0(x, y)$:

$$u_0 = \frac{1}{2} \int_{-1}^{1} f(y, z)\, dz, \quad x = -1; \quad \frac{\partial u_0}{\partial x} = 0, \quad x = 1; \quad \frac{\partial u_0}{\partial y} = 0, \quad y = \pm\delta. \tag{14}$$

The equations for the higher approximations for the mean functions $u_k(x, y)$ will differ from (10) and (14) only in the right sides, which will contain quantities calculated in the previous approximations.

Boundary-layer construction for the inhomogeneous case is substantially more complicated than that for the homogeneous case; the most severe difficulties arise in solving the problem for the ends $\bar{x} = \pm\bar{a}$ parallel to the generators of the cylinders. We illustrate the behavior of the solution near these boundaries by means of an example that allows of exact solution for rods of rectangular cross section in contact (Fig. 2).

We introduce the following notation: ξ_i ($i = 0, 1, 2\ldots$), which are the coordinates of the boundaries between the adjacent rectangles, while v_i is the value of the boundary-layer function v (the subscript related to the number of the approximation is omitted), for the range $\xi_{i-1} \leq \xi \leq \xi_i$; the equations and boundary conditions take the form

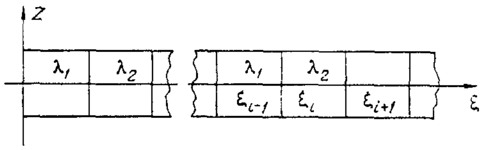

Fig. 2. A region in terms of boundary-layer variables.

$$\frac{\partial^2 v_i}{\partial \xi^2} + \frac{\partial^2 v_i}{\partial z^2} = 0,\ v_i \to 0,\quad i \to \infty,\quad \frac{\partial v_i}{\partial z} = 0,\quad z = \pm 1,$$
$$v_i = v_{i+1},\quad \lambda_1 \frac{\partial v_i}{\partial \xi} = \lambda_2 \frac{\partial v_{i+1}}{\partial \xi},\ \xi = \xi_i,\quad v = \tilde{f}(z),\ \xi = 0. \tag{15}$$

The function $\tilde{f}(z)$ appearing in the boundary condition for the end differs from the right side of the first condition in (2) by a constant appearing on the right in (14); specification of the boundary condition in this form ensures that the boundary-layer solution dies away at infinity.

The solution to (15) derived by Fourier's method is put as

$$v_i = \sum_{n=1}^{\infty} \left[D_n^{(i)} \operatorname{sh} \frac{\pi n (\xi - \xi_i)}{2} + D_n^{(i+1)} \operatorname{ch} \frac{\pi n (\xi - \xi_{i-1})}{2} \right] \cos \frac{\pi n (z+1)}{2}.$$

The matching conditions on the lines $\xi = \xi_i$ ($i = 1, 2, \ldots$) are met to generate an infinite system of the form

$$\begin{gathered} \lambda_1 D_n^{(i)} + \lambda_1 S D_n^{(i+1)} - \lambda_2 C D_n^{(i+2)} = 0, \\ C D_n^{(i+1)} + S D_n^{(i+2)} - D_n^{(i+3)} = 0, \\ \lambda_2 D_n^{(i+2)} + \lambda_2 S D_n^{(i+3)} - \lambda_1 C D_n^{(i+1)} = 0, \\ C D_n^{(i+3)} + S D_n^{(i+4)} - D_n^{(i+5)} = 0,\ i = 1,\ 2,\ \ldots;\ n = 1,\ 2,\ \ldots, \\ S = \operatorname{sh} 0.25\, \pi n t,\quad C = \operatorname{ch} 0.25\, \pi n t. \end{gathered} \tag{16}$$

For each n we obtain a system of equations with a three-diagonal matrix, namely, a second-order difference equation with periodic coefficients. This equation is solved by a method analogous to the characteristic-parameter method used for differential equations [9]. We see a solution that satisfies the occurrence relation

$$D_n^{(l)} = \mu D_n^{(l-4)},\quad l = 4,\ 5,\ 6,\ \ldots \tag{17}$$

We substitute (17) into (16) and find that a solution to (17) exists if the following condition is met:

$$\begin{gathered} D_n^{(0)} + \lambda_1 S D_n^{(1)} - \lambda_2 C D_n^{(2)} = 0,\quad C D_n^{(1)} + S D_n^{(2)} - D_n^{(3)} = 0, \\ \lambda_2 D_n^{(2)} + \lambda_2 S D_n^{(3)} - \mu \lambda_1 C D_n^{(0)} = 0,\quad C D_n^{(3)} + \mu S D_n^{(0)} - \mu D_n^{(1)} = 0. \end{gathered} \tag{18}$$

A nontrivial solution to the homogeneous system of (18) exists if

$$\Delta = \mu^2 - \mu (\lambda_1 \lambda_2 C^2)^{-1} (2\lambda_1 \lambda_2 S^2 + \lambda_1^2 S^2 + C^2 + \lambda_1 \lambda_2 S^4 + \lambda_2^2 C^2 S^2 + \lambda_1 \lambda_2) + 1 = 0. \tag{19}$$

If the solution is to be bounded as $i \to \infty$, there must be a value of μ less than 1 in magnitude; it is clear that such a value exists, for the product of the roots of (19) is 1, while the discriminant

$$D = (2\lambda_1 \lambda_2 S^2 + \lambda_1^2 S^2 C^2 + \lambda_1 \lambda_2 S^4 + \lambda_1 \lambda_2 C^4 + \lambda_2^2 C^2 S^2 + \lambda_1 \lambda_2)^2 - 4\lambda_1 \lambda_2 C^4$$

is a positive-definite quadratic form in λ_1 and λ_2. We select root μ_1 of (19) such that μ_1, $|\mu_1| < 1$ and express the constants $D_n^{(2)}$, $D_n^{(3)}$, $D_n^{(4)}$ in terms of $D_n^{(0)}$, while the constants $D_n^{(0)}$ are found from the boundary conditions for $\xi = 0$, after which (17) gives a solution for the entire region.

NOTATION

$\bar{x}$, $\bar{y}$, $\bar{z}$	are the dimensional coordinates;
x, y, z, ξ, η	are the dimensional coordinates;
u	is the dimensionless temperature;
ε	is the small parameter;

λ_1, λ_2	are the thermal conductivities;
A, B, δ, m,	
β, t, S_1, S_2	are the dimensionless constants;
v, w	are the boundary-layer functions;
λ_x, λ_y	are the effective thermal conductivities.

LITERATURE CITED

1. K. O. Friedrichs and R. F. Dressler, Comm. Pure Appl. Math., 16, No. 1, 1 (1961).
2. A. L. Gol'denbeizer, Prikl. Mat. Mekh., 26, No. 4 (1961).
3. V. V. Ponyatovskii, in: Researchers on Elasticity and Plasticity [in Russian], No. 9, Izd. LGU, Leningrad (1973), p. 81.
4. J. Cowle, Perturbation Methods in Applied [Russian translation], Mir, Moscow (1972), pp. 92, 215.
5. I. E. Zino and Yu. A. Sokovishin, Inzh.-Fiz. Zh., 26, No. 2 (1974).
6. I. E. Zeno and É. A. Tropp, Inzh.-Fiz. Zh., 28, No. 5 (1975).
7. M. G. Dzhavadov, Diff. Urav., 4, No. 10 (1968).
8. V. V. Bolotin, Mekh. Polim., No. 1 (1975).
9. B. P. Demidovich, Lectures on the Mathematical Theory of Stability [in Russian], Nauka, Moscow (1967), p. 183.

IDENTIFICATION OF TIME-VARIABLE COEFFICIENTS OF HEAT TRANSFER BY SOLVING A NONLINEAR INVERSE PROBLEM OF HEAT CONDUCTION

Yu. M. Matsevityi, V. A. Malyarenko, and A. V. Multanovskii

UDC 536.532

The solution of the inverse nonstationary problem of nonlinear heat conduction by using the method of optimal dynamic filtering is considered.

The solution of the inverse heat-conduction problem has lately assumed especially great importance, since one has to determine the boundary conditions of the heat transfer from the limited information on the temperature field of the body.

In [1, 2] the feasibility of electrical modelling of converse problems was considered; several approaches have been suggested for solving such problems on various analog models. With this aim in mind, the application of optimal dynamic filtering [3] is of some interest; it provides the possibility, as seen from previous investigations [4, 5], of solving a wide class of inverse heat-conduction problems, including the reconstruction of the temperature field, the determination of the boundary conditions, the restoring of the initial distributions, etc.

In this article a technique that enables one to obtain in a special way a prediction of the estimate of the state vector is proposed. The employed discrete-filtering algorithm of Kalman presupposes that an extended state vector can be estimated due to the specific shape of the solution of the inverse problem, in which side by side with the reconstruction of the temperature field, the identification of the boundary conditions is carried out. In view of the latter, the components of the temperature field vector and the identifying vector of parameters α are included in the state vector.

To construct a solution algorithm of the inverse problem a mathematical model was adopted by us in which the finite-differences equation of heat conduction in its matrix form as well as the identifying parameter α as a function of time are included:

Institute of Mechanical Engineering, Academy of Sciences of the Ukrainian SSR, Khar'kov.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 3, pp. 505-509, September, 1978. Original article submitted July 25, 1977.